Theory of Wing Rock

Chung-Hao Hsu* and C. Edward Lan†
University of Kansas, Lawrence, Kansas

A new nonlinear aerodynamic mathematical model for calculating wing rock characteristics is developed. The resulting nonlinear flight dynamics equations of both one and three degrees of freedom are solved with the Beecham-Titchener asymptotic method for the limit-cycle amplitude and wing rock frequency. The latter are calculated iteratively in combination with a nonlinear aerodynamic method. Reasonable agreement between theoretical and experimental results is obtained for highly swept slender wings.

Nomenclature

A = amplitude of the limit cycle in roll oscillations, deg or rad
$Ⱥ$ = wing aspect ratio, $= b^2/S$
a = amplitude of roll oscillations, deg or rad
b = wing span, ft
C_L = lift coefficient
C_ℓ = rolling moment coefficient
C_{ℓ_0} = rolling moment coefficient at zero sideslip
C_{ℓ_p} = roll damping coefficient
$C_{\ell_{p0}}$ = roll damping coefficient at zero sideslip
$C_{\ell_{pp}}$ = dimensionless variation of roll damping derivative with roll rate, $= \partial C_{\ell_p}/\partial \bar{p}$
$C_{\ell_{p\beta}}$ = dimensionless variation of roll damping derivative with sideslip, $= \partial C_{\ell_p}/\partial \beta$
$C_{\ell_{pt}}$ = total or effective roll damping coefficient
$C_{\ell_r}, C_{\ell_\beta}$ = variation of rolling moment coefficient with yaw rate and sideslip, respectively
C_n = yawing moment coefficient
C_{n_0} = yawing moment coefficient at zero sideslip
$C_{n_p}, C_{n_{p0}}$ = variation of yawing moment coefficient with roll rate and roll rate at zero sideslip, respectively
$C_{n_{pp}}$ $= \partial C_{n_p}/\partial \bar{p}$
$C_{n_{p\beta}}$ $= \partial C_{n_p}/\partial \beta$
C_{n_r}, C_{n_β} = variation of yawing moment coefficient with yaw rate and sideslip, respectively
C_Y = side-force coefficient
C_{Y_0} = side-force coefficient at zero sideslip
$C_{Y_p}, C_{Y_r}, C_{Y_\beta}$ = variation of side-force coefficient with roll rate, yaw rate, and sideslip, respectively
G = amplitude ratio of β mode to ϕ mode at limit-cycle conditions
g = acceleration of gravity, ft/s/s
H = amplitude ratio of ψ mode to ϕ mode at limit-cycle conditions
$\bar{I}$ $= (1 - I^2_{xz}/I_{xx}I_{zz})^{-1}$
I_{xx}, I_{zz} = moments of inertia about X and Z axes, slug-ft^2
I_{xy}, I_{xz}, I_{yz} = products of inertia in XYZ system, slug-ft^2
L_0 $= \bar{q}SbC_{\ell_0}/I_{xx}$, s^{-2}
L_{p0} $= \bar{q}Sb^2C_{\ell_{p0}}/2I_{xx}V$, s^{-1}
L_{pp} $= \bar{q}Sb^3C_{\ell_{pp}}/4I_{xx}V^2$, dimensionless
$L_{p\beta}$ $= \bar{q}Sb^2C_{\ell_{p\beta}}/2I_{xx}V$, s^{-1}
L_β $= \bar{q}Sbc_{\ell_\beta}/I_{xx}$, s^{-2}
L_r $= (\bar{q}Sb^2C_{\ell_r}/2I_{xx}V)\cos\alpha_s$, s^{-1}
M_∞ = freestream Mach number
m = mass, slug
N_0 $= \bar{q}SbC_{n_0}/I_{zz}$, s^{-2}
N_{p0} $= \bar{q}Sb^2C_{n_{p0}}/2I_{zz}V$, s^{-1}
N_{pp} $= \bar{q}Sb^3C_{n_{pp}}/4I_{zz}V^2$, dimensionless
$N_{p\beta}$ $= \bar{q}Sb^2C_{n_{p\beta}}/2I_{zz}V$, s^{-1}
N_r $= (\bar{q}Sb^2C_{n_r}/2I_{zz}V)\cos\alpha_s$, s^{-1}
N_β $= \bar{q}SbC_{n_\beta}/I_{zz}$, s^{-2}
P = period of the limit cycle in roll oscillations, s
$\bar{p}$ = dimensionless reduced roll rate, $pb/2V$
p = roll rate about the X axis, rad/s
$\bar{q}$ = freestream dynamic pressure, lb/ft^2
Re = Reynolds number
r = yaw rate about the Z axis, rad/s
S = wing area, ft^2
S_t = area of vertical tail, ft^2
V = magnitude of flight velocity, ft/s
X, Y, Z = rectangular coordinates of a body-fixed axis system, ft
$X_{\text{c.g.}}$ = moment-center location, ft
Y_0 $= \bar{q}SC_{Y_0}/mV$, s^{-1}
Y_p $= \bar{q}SbC_{Y_p}/2mV^2$, dimensionless
Y_r $= \bar{q}SbC_{Y_r}/2mV^2$, dimensionless
Y_β $= \bar{q}SC_{Y_\beta}/mV$, s^{-1}
α = angle of attack, deg or rad
α_{onset} = angle of attack for wing rock onset
α_s = steady-state angle of attack, deg or rad
β = sideslip angle, deg or rad
δ, ϵ = phase angle of ψ or β mode, respectively, deg or rad
η = amplitude ratio of ψ mode to ϕ mode
θ = pitch attitude angle, deg or rad
Λ_{LE} = leading-edge sweep angle, deg or rad
ξ = amplitude ratio of β mode to ϕ mode
ϕ = bank or Euler roll angle, deg or rad
ψ = heading angle, deg or rad
Ω = circular frequency of limit cycle in roll oscillations, rad/s

Superscripts

$(\dot{\ })$ = derivative with respect to time
$(\bar{\ })$ = dimensional aerodynamic derivatives defined in Eq. (15)

Introduction

WING rock is one type of lateral-directional instability for airplanes flying at high angles of attack. It occurs not only to low-aspect-ratio configurations, such as the F-4, F-5,

Presented as Paper 85-0199 at the AIAA 23rd Aerospace Sciences Meeting, Reno, NV, Jan. 14-17, 1985; received Feb. 8, 1985; revision received June 4, 1985.

*Research Engineer. Member AIAA.
†Professor of Aerospace Engineering. Associate Fellow AIAA.

F-14, X-29A, Gnat, Harrier, HP 115, and some re-entry vehicles, but also to high-aspect-ratio configurations, such as the Vari-Eze and some general-aviation airplanes. Based on extensive examination of available data,[1] it can be concluded that wing rock is triggered by flow asymmetries, developed by negative roll damping, and sustained by nonlinear aerodynamic roll damping.

Currently there are several theoretical models proposed to describe wing rocking. Each model has been verified only for a particular configuration. In fact, none of the models can predict the amplitude and period of wing rocking for all configurations. The models are described as follows.

First, it was shown that by including a cubic term in the roll damping derivative of a Gnat aircraft,[2] the angle of sideslip during wing rock could be fairly well predicted by reducing the three-degree-of-freedom (DOF) lateral-directional equations of motion to a fourth-order differential equation in β and by obtaining solutions from the application of the Beecham-Titchener method.[3] When similar cubic terms were introduced in the static yawing moment derivative due to sideslip for the HP 115 research aircraft,[4,5] the estimated amplitudes of limit cycle were about 40% greater than those obtained from six-DOF nonlinear simulations. However, using the nonlinear stiffness alone in a one-DOF model cannot explain the existence of wing rock.

Second, nonzero static lateral-directional aerodynamic forces and moments at zero sideslip angle, together with aerodynamic hysteresis, were used to reproduce the wing rock of an F-5E in a six-DOF digital computer simulation.[6] The aerodynamic hysteresis of the rolling moment with sideslip was also assumed in a simple two-DOF model including rolling and yawing moments to demonstrate that roll hysteresis could be a potential cause of wing rock.[7] Mathematically, such hysteresis can be accounted for by including a function that can assign two possible values in the rolling moment for a given sideslip angle. A single-DOF model with a hysteresis loop can yield the motion of wing rock. However, limit cycles are obtained only when an external disturbance is large enough to induce a sideslip angle to lie outside of the β range in the hysteresis loop. Furthermore, the loop shape must be determined before solving the equation of motion.

Third, the variation of roll damping with sideslip angle, such that damping was negative at small sideslip angles but positive at large sideslip angles, could induce wing rock for a delta wing 80-deg sweep.[8] A one-DOF nonlinear simulation for this delta wing produced the motion of wing rock in close agreement with test results of a free-to-roll model. However, it was also shown that an approximate analytical solution of the one-DOF equation based on the Beecham-Titchener method by assuming a linear variation of roll damping with sideslip angle underestimated the amplitudes of wing rock by 15% for angles of attack between 25 and 35 deg.

Fourth, by representing the time-history effect by a lumped time lag,[9] limit-cycle amplitudes were predicted using experimental static data. However, this one-DOF analysis could predict only the limit-cycle amplitude for delta wings with leading-edge sweep larger than 74 deg. In addition, the frequency must be known in advance.

Finally, the unsteady incompressible inviscid flow equations and the one-DOF equation of motion were solved simultaneously,[10] resulting in dynamic hysteresis of rolling moment vs bank angles. The method is expected to be quite time consuming.

In this study, an appropriate nonlinear aerodynamic model is developed to investigate the main aerodynamic nonlinearities causing wing rock. By applying the method of Ref. 3 to the one- and three-DOF flight dynamics equations, approximate expressions for the limit-cycle amplitude and period of wing rocking are obtained.

Theoretical Formulation

Wing rock is an uncommanded roll-yaw oscillation dominated by roll motion oscillating with a constant amplitude. It may be initiated either with a sideslip or during a zero-sideslip flight with some flow asymmetries over the aircraft flying at high angles of attack. Once the asymmetric flow starts, a roll-oscillation amplitude will keep building up if the roll damping is negative. The transient amplitude of wing rock will grow gradually over some oscillation cycles because of roll instability and negligible dihedral effect at low roll angle.

Although the roll damping is negative at small roll angles, it is positive at larger roll angles for a sustained wing rock. Both the effective dihedral effect and positive roll damping via aerodynamic nonlinearities at large roll angles will gradually reduce the roll rate. As these restoring moments become stronger, the aircraft will reach a threshold roll angle and finally switch the rolling direction. Thus, wing rock is constrained to a finite-amplitude oscillation through nonlinear roll damping.

To describe the aforementioned observed phenomena mathematically, dynamic equations of motion and their approximate solutions are developed for both the one- and three-DOF cases.

One Degree of Freedom

Based on a body-fixed axis system, the equation of motion can be written as

$$(I_{xx}/\bar{q}Sb)\dot{p} = C_\ell(t) \tag{1}$$

where I_{xx} is the rolling moments of inertia, $\bar{q}$ the dynamic pressure, S the reference wing area, b the wing span, $\dot{p}$ the roll angular acceleration, and $C_\ell(t)$ the total aerodynamic rolling moment coefficient.

Based on the test data[8] for a delta wing of 80-deg sweep, the total rolling moment coefficient can be written as

$$C_\ell(t) = C_{\ell_0} + C_{\ell_\beta}\beta + C_{\ell_{pt}}\bar{p} \tag{2}$$

where

$$C_{\ell_{pt}} = C_{\ell_{p0}} + C_{\ell_{p\beta}}\,|\beta| + C_{\ell_{pp}}\,|\bar{p}| \tag{3}$$

In Eq. (3) the second derivative, $C_{\ell_{pp}}$, is needed because test data[8] show that C_{ℓ_p} is a function of $\bar{p}$. It should be noted that in Eq. (3) the absolute sign on β and $\bar{p}$ is very important. In fact, if both $|\beta|$ and $|\bar{p}|$ are to be developed in polynomials, an infinite number of terms would be needed, at least theoretically. A similar situation can be found in Ref. 11, p. 136. For comparison, a fifth-degree polynominal,[10] in terms of roll angle, was used to fit the rolling moment through the least-square method.

Equation (1), together with the kinematic relationships

$$p = \dot{\phi}, \quad \beta \simeq \phi \sin\alpha_s \tag{4}$$

can be reduced to the following equation with coefficients in terms of dimensional derivatives.

$$\ddot{\phi} = L_0 + \sin\alpha_s L_\beta \phi + (L_{p0} + \sin\alpha_s L_{p\beta}\,|\phi| + L_{pp}\,|\dot{\phi}|)\dot{\phi} \tag{5}$$

To solve this nonlinear second-order differential equation for a steady-state limit-cycle wing rock, it is assumed that

$$\phi(t) = a(t)\cos\nu(t) \tag{6}$$

Substituting Eq. (6) into Eq. (5) and assuming that the frequency ν and amplitude a do not vary greatly over one oscillation cycle, the Beecham-Titchener method[3] allows Eq. (5) to be decomposed into two coupled nonlinear first-order differential equations. The circular frequency Ω, period P, and amplitude A of the limit cycle can be obtained directly from these coupled nonlinear first-order equations. To the first-order approximation, they are given as follows[1]:

$$\Omega = (-\sin\alpha_s L_\beta)^{1/2}$$
$$= [-(\bar{q}Sb/I_{xx})\sin\alpha_s C_{\ell_\beta}]^{1/2} \tag{7}$$

$$P = 2\pi/\Omega \tag{8}$$

and

$$A = -(3\pi/4)L_{p0}/(\sin\alpha_s L_{p\beta} + 2\Omega L_{pp})$$
$$= -(3\pi/4)C_{\ell_{p0}}/[\sin\alpha_s C_{\ell_{p\beta}} + (\Omega b/V)C_{\ell_{pp}}] \tag{9}$$

Equation (9) is reduced to the expression used in Ref. 8 if $C_{\ell_{pp}}$ is set to zero.

Three Degrees of Freedom

Available data[12] indicate that the roll amplitude is at least one order of magnitude larger than the yaw amplitude in wing rock conditions. Therefore, it is assumed that

$$p \simeq \dot{\phi} \tag{10}$$

$$r \approx \dot{\psi}\cos\alpha_s \tag{11}$$

Using Eqs. (2) and (3) for the rolling moment coefficient and a similar expression for the yawing moment coefficient, the lateral-directional equations of motion can be written as

$$\dot{\beta} = \bar{Y}_0 + \bar{Y}_\beta\beta + \bar{Y}_p\dot{\phi} + \bar{Y}_r\dot{\psi} + \bar{Y}_\phi\phi \tag{12}$$

$$\ddot{\phi} = \bar{L}_0 + \bar{L}_\beta\beta + (\bar{L}_{p0} + \bar{L}_{p\beta}|\beta| + \bar{L}_{pp}|\dot{\phi}|)\dot{\phi} + \bar{L}_r\dot{\psi} \tag{13}$$

$$\ddot{\psi} = \bar{N}_0 + \bar{N}_\beta\beta + (\bar{N}_{p0} + \bar{N}_{p\beta}|\beta| + \bar{N}_{pp}|\dot{\phi}|)\dot{\phi} + \bar{N}_r\dot{\psi} \tag{14}$$

where

$$\begin{aligned}
\bar{Y}_0 &= \bar{Y}_0, & \bar{Y}_\beta &= Y_\beta \\
\bar{Y}_p &= Y_p + \sin\alpha_s, & \bar{Y}_r &= (Y_r - \cos\alpha_s)\cos\alpha_s \\
\bar{Y}_\phi &= (g/V)\cos\theta, & \bar{I} &= (1 - I_{xz}^2/I_{xx}I_{zz})^{-1} \\
\bar{L}_0 &= \bar{I}(L_0 + I_{xz}N_0/I_{xx}), & \bar{L}_\beta &= \bar{I}(L_\beta + I_{xz}N_\beta/I_{xx}) \\
\bar{L}_{p0} &= \bar{I}(L_{p0} + I_{xz}N_{p0}/I_{xx}), & \bar{L}_{p\beta} &= \bar{I}(L_{p\beta} + I_{xz}N_{p\beta}/I_{xx}) \\
\bar{L}_{pp} &= \bar{I}(L_{pp} + I_{xz}N_{pp}/I_{xx}), & \bar{L}_r &= \bar{I}(L_r + I_{xz}N_r/I_{xx}) \\
\bar{N}_0 &= \bar{I}(I_{xz}L_0/I_{zz} + N_0)/\cos\alpha_s, & \bar{N}_\beta &= \bar{I}(I_{xz}L_\beta/I_{zz} + N_\beta)/\cos\alpha_s \\
\bar{N}_{p0} &= \bar{I}(I_{xz}L_{p0}/I_{zz} + N_{p0})/\cos\alpha_s, & \bar{N}_{p\beta} &= \bar{I}(I_{xz}L_{p\beta}/I_{zz} + N_{p\beta})/\cos\alpha_s \\
\bar{N}_{pp} &= \bar{I}(I_{xz}L_{pp}/I_{zz} + N_{pp})/\cos\alpha_s, & \bar{N}_r &= \bar{I}(I_{xz}L_r/I_{zz} + N_r)/\cos\alpha_s
\end{aligned} \tag{15}$$

The solutions to Eqs. (12-14) are assumed to be of the form

$$\beta(t) = a(t)\xi(t)\cos[\nu(t) + \epsilon(t)] \tag{16}$$

$$\phi(t) = a(t)\cos\nu(t) \tag{17}$$

$$\psi(t) = a(t)\eta(t)\cos[\nu(t) + \delta(t)] \tag{18}$$

where ξ and η are instantaneous amplitude ratios of modes β and ψ to mode ϕ, and ϵ and δ are the corresponding phase angles. Again, the Beecham-Titchener method is applied to obtain the limit-cycle amplitude and frequency for a steady-state wing rock.[1] The resulting first-order approximate solutions are

Amplitude:

$$A = -(3\pi/4)(\bar{L}_{p0} + H\bar{L}_r)/(G\bar{L}_{p\beta} + 2\Omega\bar{L}_{pp})$$
$$= -(3\pi/4)(gC_{\ell_{p0}} + fC_{n_{p0}})[G(gC_{\ell_{p\beta}} + fC_{n_{p\beta}})$$
$$+ (\Omega b/V)(gC_{\ell_{pp}} + fC_{n_{pp}})] \tag{19}$$

where

$$h = (I_{zz}C_{\ell_r} + I_{xz}C_{n_r})/(I_{xz}C_{\ell_r} + I_{xx}C_{n_r})$$
$$g = 1 - h(I_{xz}/I_{zz})$$
$$f = (I_{xx}/I_{zz})(I_{xz}/I_{xx} - h)$$

Frequency:

$$\Omega^2 = -G\bar{L}_\beta$$
$$= -G\bar{q}Sb(I_{zz}C_{\ell_\beta} + I_{xz}C_{n_\beta})/(I_{xx}I_{zz} - I_{xz}^2) \tag{20}$$

where

$$G = \bar{Y}_p + H\bar{Y}_r = \sin\alpha_s - H\cos^2\alpha_s$$
$$+ (\bar{q}Sb/2mV^2)C_{Y_p} + H(\bar{q}Sb/2mV^2)C_{Y_r}\cos\alpha_s \tag{21}$$

$$H = -[\bar{N}_{p0} + (4/3\pi)A(G\bar{N}_{p\beta} + 2\Omega\bar{N}_{pp})]/\bar{N}_r$$
$$= -\{I_{xz}C_{\ell_{p0}} + I_{xx}C_{n_{p0}} + (4/3\pi)A[G(I_{xz}C_{\ell_{p\beta}}$$
$$+ I_{xx}C_{n_{p\beta}}) + (b\Omega/V)(I_{xz}C_{\ell_{pp}} + I_{xx}C_{n_{pp}})]\}$$
$$\div \{I_{xz}C_{\ell_r} + I_{xx}C_{n_r}\}\cos\alpha_s \tag{22}$$

As can be seen from Eq. (19), the roll amplitude of a limit cycle is affected not only by C_{ℓ_β}, $C_{\ell_{p0}}$, $C_{\ell_{p\beta}}$, and $C_{\ell_{pp}}$, as in the one-DOF model, but also by C_{n_β}, C_{y_p}, C_{y_r}, $C_{n_{p0}}$, $C_{n_{p\beta}}$, $C_{n_{pp}}$, C_{ℓ_r}, and C_{n_r}. Equations (19-22) are coupled nonlinear algebraic equations in terms of A, G, H, and Ω that can be solved through iterations.[1] For a steady-state wing rock, all solutions must be real and positive.

Numerical Results

To test the present theory of predicting the amplitude and period of a steady-state wing rock, wind tunnel data of dynamic stability derivatives are needed. Unfortunately, these data, in particular the second derivatives, $C_{\ell_{p\beta}}$, $C_{\ell_{pp}}$, etc., mostly are unavailable. Therefore, for the present purpose, the computer code developed in Ref. 13 will be used to estimate these derivatives. According to the Beecham-Titchener method, these derivatives should be evaluated at some average dynamic conditions. Since the interaction between aerodynamics and dynamics in wing rock is strongly nonlinear, an iterative scheme[1] is developed to calculate the average aerodynamic derivatives and dynamic characteristics for a steady-state limit cycle. In other words, these derivatives are calculated at some average dynamic conditions (i.e., certain average values of β and $\bar{p}$) which, in turn, depend on the aerodynamic characteristics. These average values of β and $\bar{p}$ satisfy the following phase-plane relation[1]:

$$\Phi^2 + \dot{\phi}^2/\Omega^2 = A^2/4 \tag{23}$$

From Eq. (23), the average bank angle ϕ_{ave}, roll rate $\bar{p}_{\text{ave}}$, and sideslip angle β_{ave} can be obtained to be

$$\phi_{\text{ave}} = A/2 \tag{24}$$

$$\bar{p}_{\text{ave}} = \dot{\phi}_{\text{ave}}b/2V = (\Omega A/2)b/2V \tag{25}$$

$$\beta_{\text{ave}} = \phi_{\text{ave}}\sin\alpha_s \tag{26}$$

The derivative $C_{\ell_{p\beta}}$ is calculated by a simple finite difference between the following two dynamic conditions: ($\beta = \beta_{\text{ave}}$, $\bar{p} = 0$) and ($\beta = 0$, $\bar{p} = \bar{p}_{\text{ave}}$). Note that both conditions satisfy

Eq. (23). The derivative $C_{\ell_{pp}}$ is calculated from the dynamic conditions: $(\beta=\beta_{ave}, \bar{p}=\bar{p}_{max})$ and $(\beta=\beta_{ave}, \bar{p}=0)$, where $\bar{p}_{max}=2\bar{p}_{ave}$.

The iterative process for the nonlinear interaction between dynamics and aerodynamics is started by choosing a proper value of β_{ave}, say, 10 deg. $\bar{p}_{ave}$ is calculated from Eq. (25) with Ω given by Eq. (7) or (20). Aerodynamic derivatives are then calculated with these dynamic conditions. These aerodynamic derivatives are then used to calculate A, and hence β_{ave}, from Eq. (9) or (19). The process is repeated until convergence is achieved. Normally three iterations are needed for convergence.

One Degree of Freedom

Calculated results for a delta wing of 80-deg sweep are compared with free-to-roll test data[8,14] in Fig. 1 for the amplitude and in Fig. 2 for the period. Figure 1 shows that the prediction of the amplitude is generally good up to $\alpha=38$ deg, beyond which a converged solution is difficult to obtain. In addition to the strong vortex breakdown effect at higher α, the viscous flow separation will probably also become important. The latter effect is not included in the computer program of Ref. 13. For this reason, the aerodynamic derivatives may not be accurate enough for wing rock calculations. It should be noted that vortex breakdown over the wing will not occur until $\alpha \approx 33$ deg for this wing in wing rock, and it will reduce wing rock amplitude as shown in Fig. 1. The predicted limit-cycle period is reasonably good, as shown in Fig. 2. A detailed analysis of calculated results indicated that the roll-rate dependency of roll damping (i.e., $C_{\ell_{pp}}$) played an important role in determining the limit-cycle amplitude.

The histograms of C_ℓ vs ϕ for one limit cycle of wing rock based on Eqs. (2) and (3) without C_{ℓ_0} at $\alpha=27$ and 32 deg are presented in Figs. 3 and 4, respectively. It is seen that Eqs. (2) and (3) can describe the dynamic hysteresis properly. Agreement of theoretical contours with the data derived from free-to-roll tests[8] is reasonable. Because of the absence of C_{ℓ_0}, the theoretical results are symmetric about positive and negative bank angles. However, the test data show some asymmetries. They are most likely due to the vortex asymmetry at high angles of attack and zero sideslip. The effect of such vortex asymmetry is not accounted for in the method of Ref. 13.

Three Degrees of Freedom

Simple flying wings with a vertical tail were extensively tested in the NACA Langley free-flight tunnel.[15] Four models having delta wings with 53, 63.5, 76, and 82.9 deg leading-edge sweep and four cropped delta wings with a taper ratio of 0.5 are used in the verification of the present theory.

Theoretical results of two flying wings are compared with data in Figs. 5 and 6. For both configurations—a 76-deg delta wing and a 76-deg cropped delta wing with a vertical tail—the predicted lift coefficients agree well with data up to $\alpha=30$ deg. The present theory overpredicts the starting α of wing rock by about 5 deg. The shaded areas in both figures represent the approximate α range in which the steady-state wing rock was observed. At α below that range, no wing rock was observed; at α above that range, the amplitude of roll oscillation kept increasing rapidly as α was increased further. The predicted α ranges of wing rock for both configurations are less than 10 deg of α. The solid lines in both figures cannot be extended to higher $-\alpha$ values because the iterative scheme yields no converged solutions or Eqs. (19-22) give negative amplitudes or imaginary frequencies.

Only five flying models of large sweep angles with an aspect ratio equal to or less than one were reported to exhibit wing rock. The theoretical prediction (Fig. 7) shows the same trend. The agreement between the predicted onset α of limit cycle and data is within ± 5 deg.

Applications of the present theory to configurations of moderate to high aspect ratios have been presented elsewhere.[16]

In the numerical analysis, it was found that the most important stability derivatives were $C_{\ell_{p0}}$, $C_{\ell_{p\beta}}$, and $C_{\ell_{pp}}$. In the one-DOF theory, it is required that $C_{\ell_{p0}}$ be positive (i.e., undamped) to cause a limit-cycle motion. However, in the three-

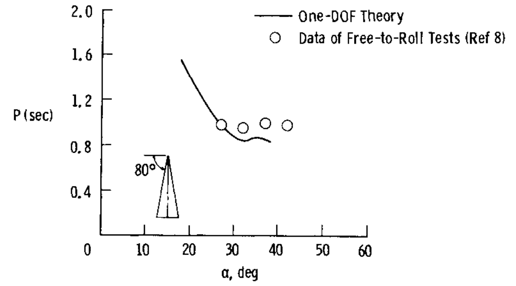


Fig. 2 Steady-state periods of wing rock for a delta wing of 80-deg sweep ($A\!\!\!/R=0.71$).

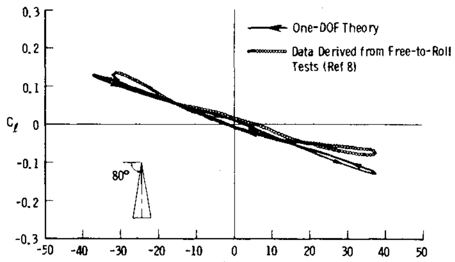


Fig. 3 Histogram of the total aerodynamic rolling moment coefficient vs bank angle for one limit cycle of wing rock at 27-deg angle of attack.

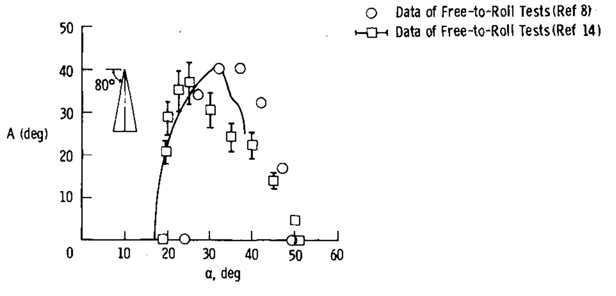


Fig. 1 Steady-state amplitude of wing rock for a delta wing of 80-deg sweep ($A\!\!\!/R=0.71$).

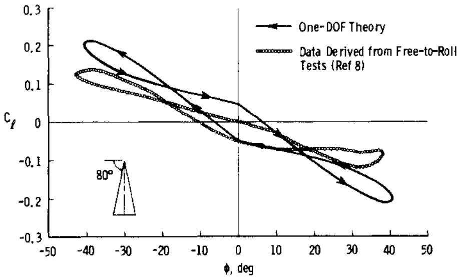


Fig. 4 Histogram of the total aerodynamic rolling moment coefficient vs bank angle for one limit cycle of wing rock at 32-deg angle of attack.

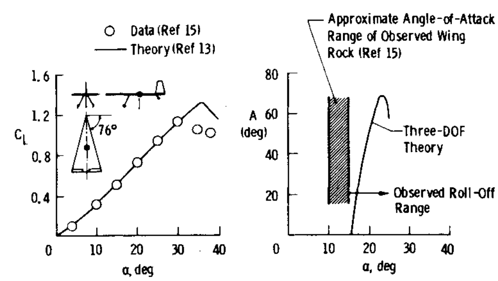


Fig. 5 Lift coefficient (data obtained at $Re = 696{,}000$) and steady-state amplitude of wing rock for a delta wing ($\mathcal{R} = 1.0$) with a vertical tail at $M_\infty = 0.1$. $X_{c.g.} = 0.30\bar{c}$.

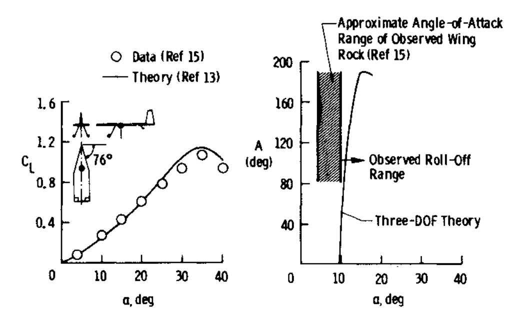
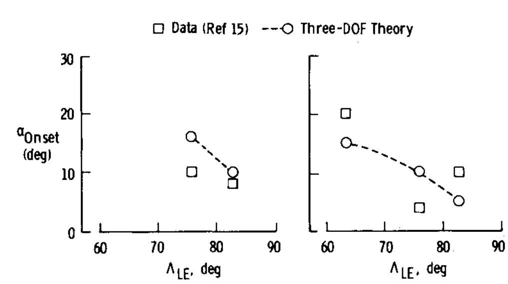


Fig. 6 Lift coefficient (data obtained at $Re = 812{,}000$) and steady-state amplitude of wing rock for a cropped delta wing of 76-deg sweep ($\mathcal{R} = 0.33$) with a vertical tail at $M_\infty = 0.1$. $X_{c.g.} = 0.27\bar{c}$.



(a) Delta wing with a vertical tail $S_t/S = 0.10$

(b) Cropped delta wing with a vertical tail $S_t/S = 0.134$

Fig. 7 Comparison of the onset α with data at $M_\infty = 0.1$. Moment-center location is at forward gravity-center position.

DOF theory, wing rock is possible even if $C_{\ell_{p0}}$ is negative but small in magnitude. The reason is that the combined effects of C_{ℓ_r}, $C_{n_{p0}}$, and C_{n_r} may make the aircraft more unstable in roll oscillations.

Conclusions

A theory based on available experimental data to calculate wing rock characteristics is presented. A new nonlinear aerodynamic model that includes all essential aerodynamic nonlinearities causing wing rock has been developed.

To determine the numerical values of the limit-cycle amplitude and frequency, the important aerodynamic derivatives such as $C_{\ell_{p0}}$, $C_{\ell_{p\beta}}$, $C_{\ell_{pp}}$, C_{ℓ_r}, $C_{n_{p0}}$, C_{n_r}, C_{ℓ_β}, and C_{n_β} are required. Because of the strong interaction between aerodynamics and dynamics during wing rocking, these derivatives are evaluated with a steady-flow aerodynamics computer code at some average dynamic conditions in an iterative manner, as explained in the section titled Numerical Results.

Theoretical results show that to sustain a steady-state wing rock, the total aerodynamic roll damping must be negative at small bank angles. On the other hand, at larger bank angles it must be positive.

Reasonable agreement between theoretical and experimental results has been obtained for many slender wings, of which all but one are equipped with a vertical tail.

Acknowledgment

This research was supported by NASA Grant NAG-1-134.

References

[1]Hsu, C. H., "Theory of Wing Rock," Ph.D. Thesis, University of Kansas, Lawrence, Feb. 1984.

[2]Ross, A. J., "Lateral Stability at High Angles of Attack, Particularly Wing Rock," Paper 10, AGARD-CP-260, Sept. 1978.

[3]Beecham, L. J. and Titchener, I. M., "Some Notes on an Approximate Solution for the Free Oscillation Characteristics of Non-Linear Systems Typified by $\ddot{x} + F(x,\dot{x}) = 0$," British ARC R&M 3651, Aug. 1969.

[4]Ross, A. J., "Investigation of Nonlinear Motion Experienced on a Slender-Wing Research Aircraft," *Journal of Aircraft*, Vol. 9, Sept. 1972, pp. 625-631.

[5]Ross, A. J. and Beecham, L. J., "An Approximate Analysis of the Nonlinear Lateral Motion of a Slender Aircraft (HP 115) at Low Speeds," British ARC R&M 3674, May 1970.

[6]Cord, T., "Hysteresis-Induced Wing Rock," AFFDL-TM-75-76-FGC, June 1975.

[7]Schmidt, L. V., "Wing Rock Due to Aerodynamic Hysteresis," *Journal of Aircraft*, Vol. 16, March 1979, pp. 129-133.

[8]Nguyen, L. T., Yip, L. P., and Chambers, J. R., "Self-Induced Wing Rock of Slender Delta Wings," AIAA Paper 81-1883, Aug. 1981.

[9]Ericsson, L. E., "The Fluid Mechanics of Slender Wing Rock," *Journal of Aircraft*, Vol. 21, May 1984, pp. 322-328.

[10]Konstadinopoulos, P., Mook, D. T., and Nayfeh, A. H., "Subsonic Wing Rock of Slender Delta Wings," *Journal of Aircraft*, Vol. 22, March 1985, pp. 223-228.

[11]Nayfeh, A. H. and Mook, D. T., *Nonlinear Oscillations*, John Wiley & Sons, NY, 1979.

[12]Nguyen, L. T. and Gilbert, W. P., "Application of High-α Control System Concepts to a Variable-Sweep Fighter Airplane," AIAA Paper 80-1582, Aug. 1980.

[13]Lan, C. E. and Hsu, C. H., "Effects of Vortex Breakdown on Longitudinal and Lateral-Directional Aerodynamics of Slender Wings by the Suction Analogy," AIAA Paper 82-1385, Aug. 1982.

[14]Levin, D. and Katz, J., "Dynamic Load Measurements with Delta Wings Undergoing Self-Induced Roll-Oscillation," *Journal of Aircraft*, Vol. 21, Jan. 1984, pp. 30-36.

[15]McKinney, M. O. Jr. and Drake, H. M., "Flight Characteristics at Low Speed of Delta-Wing Models," NACA RM L7K07, Jan. 1948.

[16]Lan, C. E., "Theoretical Prediction of Wing Rocking," AGARD Symposium on Unsteady Aerodynamics—Fundamentals and Applications to Aircraft Dynamics, Göttingen, FRG, May 1985.